Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

$$\left.\begin{array}{l} x^2-2ax\cos t+y^2-2ay\sin t+a^2+c^2t^2-R^2=0 \\ ax\sin t-ay\cos t+c^2t=0 \end{array}\right\} \quad (3)$$

or

$$\left.\begin{array}{l} (x^2+y^2)+a^2+c^2t^2-R^2=2a(x\cos t+y\sin t) \\ 2c^2t=2a(-x\sin t+y\cos t) \end{array}\right\}. \quad (4)$$

Squaring and adding, we have

$$(x^2+y^2)^2+2(x^2+y^2)(a^2+c^2t^2-R^2)+(a^2+c^2t^2-R^2)^2+4c^4t^2=4a^2(x^2+y^2)$$

a quadratic equation for x^2+y^2, or the square of the radius vector r. And if we use polar coordinates, $\left.\begin{array}{l} x=r\cos\phi \\ y=r\sin\phi \end{array}\right\}$, this equation enables us to find r. Putting for x and y, $r\cos\phi$ and $r\sin\phi$, respectively, in the last of equations (4), we have

$$c^2t=ar\sin(\phi-t), \text{ or } \phi=t+\sin^{-1}\frac{c^2t}{ar}.$$

The equations of the curve are then

$$r=\sqrt{[2\sqrt{(a^2R^2-a^2c^2t^2-c^4t^2)}-(c^2t^2-a^2-R^2)]}, \quad \phi=t+\sin^{-1}\frac{c^2t}{ar}.$$

Also solved by G. B. M. Zerr. An incomplete solution was received from the Proposer.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

145. Proposed by J. D. WILLIAMS, being the 12th of his fourteen challenge problems proposed in 1832.

Make $x^2+y^2=\square$, $\frac{5}{4}(x^2+y^2)$=a cube, $xy=2x^3$, $2(x+y)+\frac{xy}{x+y}=\square$, and $(x^4+y^4)(x^2+y^2)-(x^5+y^5)\sqrt{(x^2+y^2)}=\square$.

Solution by DR. E. SWIFT, Princeton University.

If $x^2+y^2=\square$, we must have $x=2\xi\eta k$, $y=(\xi^2-\eta^2)k$, where ξ, η, k are integers. Then $x^2+y^2=k^2[\xi^2+\eta^2]^2$,

If $\frac{5}{4}$ of this=a cube, evidently it must be of the form $4\times25\times$a cube, or $k(\xi^2+\eta^2)$ is of the form $2\times5\times$a cube.

Since $xy=2x^3$, either, a) $x=0$, or, b) $y=2x^2$.

If a) is true, $y=\sqrt{(x^2+y^2)}$ and must be of the form $2\times5\times$a cube.

$x=0$ satisfies the last condition; it remains to satisfy the third, which reduces to $2y=\square$

Since $y=10a^3$, the least value of a which makes $2y^2$ square is 5, and $y=10\times125=1250$.

The values $x=0$, $y=1250$ are the smallest values satisfying the conditions when $x=0$. The complete solution is $x=0$, $y=1250a^6$, a being any number.

If $x\neq 0$, $y=2x^2$, and we must have, putting in the values of x and y in terms of ξ and η,

$$(\xi^2-\eta^2)k=4\xi^2\eta^2k^2, \text{ or } \xi^2-\eta^2=4k\xi^2\eta^2,$$

a second relation between ξ and η. But this is evidently impossible, if ξ and η have integral values, as is necessary if x and y are to be integral; for transposing $\xi^2=\eta^2(4k\xi^2+1)>\xi^2$. The only *integral* solutions, then, are $x=0$, $y=1250a^6$.

147. **Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.**

If $4n+3$ is prime, $2(1.2.3...4n)+1\equiv 0 \pmod{4n+3}$; and conversely. If $4n+3$ is prime, $(1.2.3...2n)^2-4\equiv 0 \pmod{4n+3}$; and conversely.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

By Wilson's Theorem, $1+(4n+2)!\equiv 0 \pmod{4n+3}$.

$\therefore 1+(4n+2)(4n+1)(4n)!\equiv 0 \pmod{4n+3}$;

$1+[4n(4n+3)+2](4n)!\equiv 0 \pmod{4n+3}$.

$\therefore 1+2(1.2.3.4...4n)\equiv 0 \pmod{4n+3}$.

By a corollary to Wilson's Theorem, $[(2n+1)!]^2-1\equiv 0 \pmod{4n+3}$.

$\therefore (2n+1)^2[(2n)!]^2-1\equiv 0 \pmod{4n+3}$;

$[n(4n+3)+n+1][(2n)!]^2-1\equiv 0 \pmod{4n+3}$.

$\therefore 4(n+1)[(2n)!]^2-4\equiv 0 \pmod{4n+3}$.

$\therefore (4n+3)[(2n)!]^2+[(2n)!]^2-4\equiv 0 \pmod{4n+3}$.

$\therefore (1.2.3.4...2n)^2-4\equiv 0 \pmod{4n+3}$.

The converse follow at once.

AVERAGE AND PROBABILITY.

192. **Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.**

A point is taken at random in a square whose side is $2a$. With this point as center and radius$=a$ a circumference is described. What is the mean area of that part of the circle which lies within the square?

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

By considering the square AO, $\frac{1}{4}$ the original square AC, all possible positions of the circle are taken. Let P, Q be the center of the circle, P taken in the quadrant of a circle radius a, center A, and Q taken in AO